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THREE-FAMILY MASS MATRICES LEADING TO A VERY MASSIVE TOP QUARK

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Abstract

We study 3-family mass matrices which can lead to a top quark of mass greater than 100 GeV. All known mixing data can be satisfied with top masses as large as 200 GeV with standard minimal Higgs structure, but only as large as 125 GeV in the presence of two Higgs doublets with equal vacuum expectation values and a charged Higgs mass of 50 GeV. A particularly simple set of mass matrices is found which has the phase structure proposed by Shin for the Fritzsch set of matrices and leads to a top quark mass of 135 GeV. Comparison is made with the superstring form suggested by Casas and Muñoz.

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In lieu of a well-defined dynamical theory of mass generation, much work[1] has gone into the study of quark mass matrices with the goal of extracting useful information concerning bounds on the top quark mass and the number of families required to obtain good fits to the known charged-current and higher-order neutral-current mixing data. Since it now appears that the number of standard families with ultralight neutrinos is limited to three,[2] once the top quark is discovered and its mass is pinned down, one can in turn invert the procedure and hope to extract useful information on the dynamical chiral symmetry-breaking mechanism itself.

The mass matrices have also served as a good testing ground for certain hypotheses. In particular, Fritzsch[3] proposed twelve years ago that the quark masses are generated by a hierarchical sequence of chiral symmetry breaking with only nearest-neighbor interactions allowed, presumably by some discrete symmetries. Although this idea is very attractive, it could only be severely tested in the past year once information about the small KM mixing elements became more accurately determined, especially for V_{td} and V_{ub}. The $B_d - \bar{B}_d$ mixing data require that at least two Higgs doublets be present for the Fritzsch model to survive,[4] but new information on $V_{ub}/V_{cb} \simeq 0.10 \pm 0.03$ from the observed B meson semileptonic decay spectrum[5] tends to disfavor the model in its original form since the prediction[4] is only in the range 0.05 - 0.06. Modifications to the Fritzsch hypothesis were suggested by Lindner and the author[6] which raised the ratio to $|V_{ub}/V_{cb}| \sim 0.14$. But in either version, the author showed[7] that the top mass is bounded to be less than ~ 100 GeV.

Since the number of standard families appears to be limited to three from the observed width of the weak Z boson,[2] and the top quark mass has already been bounded[8] to be greater than 78 GeV and may well be larger than 100 GeV, a critical reexamination of three-family quark mass matrices appears to be in order. By studying rank two matrices where the up and down quark masses are set equal to zero, we shall demonstrate that top quark masses greater than 100 GeV are possible, provided a non-zero diagonal entry in the 22 element of down matrix is allowed. In fact, we have carried out an extensive search of rank three matrices and find top quark masses as large as 200 GeV are possible with standard Higgs structure, whereby all the known mixing parameters are fit reasonably well. This limit is not quite as large as 230 GeV preferred by Bardeen, Hill and Lindner[9] on the basis of a minimal dynamical symmetry breaking of the standard model. If two Higgs doublets are present, however, the top mass bound is lowered considerably. For example, with equal vacuum expectation values (VEV's) and a charged Higgs scalar mass of 50 GeV (90 GeV), the upper limit on the top quark mass appears to be only 125 GeV (135 GeV).

We have also searched for simple forms for the mass matrices and, in fact, find one which has the phase structure originally proposed by Shin[10] for the Fritzsch model. That form was ruled out several years ago in the Fritzsch case,[4] but in our version we find it leads to a top mass of approximately 135 GeV. We shall also compare the forms of the mass matrices obtained with a heavy top quark to that favored by Casas and Muñoz[11] in the superstring framework.

In order to determine an acceptable set of mass matrices which satisfactorily fit the data and to compare directly with our earlier work, we shall use the Gasser-Leutwyler set of quark masses[12] defined at 1 GeV for $\Lambda_3 = 100$ MeV

$$\begin{aligned} m_u &= 5.1 \pm 1.5 \text{ MeV}, & m_d &= 8.9 \pm 2.6 \text{ MeV} \\ m_c &= 1.35 \pm 0.05 \text{ GeV}, & m_s &= 175 \pm 55 \text{ MeV} \\ m_t &= ? & m_b &= 5.3 \pm 0.1 \text{ GeV} \end{aligned} \tag{1}$$

and the 3-family KM matrix determination of Schubert,[13]

$$(|\mathbf{V}_{ij}|) = \begin{pmatrix} 0.9754 \pm 0.0004 & 0.2206 \pm 0.0018 & 0.0000 + 0.0123 \\ 0.2203 \pm 0.0019 & 0.9743 \pm 0.0005 & 0.0460 \pm 0.0060 \\ 0.0101 \pm 0.0122 & 0.0449 \pm 0.0065 & 0.9989 \pm 0.0003 \end{pmatrix} \tag{2}$$

To achieve as large top masses as possible, we select $m_s = 120$ MeV on the low side along with $m_u = 3.5$ MeV and $m_d = 6.1$ MeV, so as to maintain the more accurately known light quark mass ratios derived from current algebra. Other useful experimentally determined quantities are the Jarlskog J-value[14] which is a measure of CP violation in the KM matrix

$$J = (3.0 \pm 0.5) \times 10^{-5} \tag{3}$$

the bag parameter[15] in K decay which enters the CP-violating ϵ parameter,

$$B_K = 0.55 - 0.90 \tag{4}$$

and the $B_d - \bar{B}_d$ mixing parameter[5]

$$x_d = 0.66 \pm 0.09 \tag{5a}$$

which leads to the combination

$$m_t^2 |V_{td}^* V_{tb}|^2 R \simeq (1.8 \pm 0.3) \frac{(0.140)^2}{B_B f_B^2} \tag{5b}$$

appearing in the box diagrams with W exchange and also charged scalar Higgs exchange in the two-doublet Higgs model. The factor R represents correction factors depending on the running top quark mass and is typically less than unity with standard Higgs structure, but greater than unity with the inclusion of charged Higgs exchange as the interference effect is constructive.[4] Due to the theoretical uncertainty in the B-meson decay constant, f_B, and bag parameter, B_B, we allow the righthand side of (5b) to assume the range

$$m_t^2 |V_{td}^* V_{tb}|^2 R \simeq 1.3 - 2.3 \tag{5c}$$

As noted earlier, the recent $|V_{ub}/V_{cb}|$ ratio determined[5] from the B-meson semileptonic decay spectrum is quoted to be $|V_{ub}/V_{cb}| \simeq 0.10 \pm 0.03$ although the extraction of this number is somewhat model-dependent, and we shall allow the more generous range

$$|V_{ub}/V_{cb}| \simeq 0.07 - 0.15 \tag{6}$$

The recent measurements[16] of the direct CP violation parameter, ϵ'/ϵ, by the NA31 group at CERN and the E731 group at FNAL

$$\epsilon'/\epsilon = \begin{cases} (33 \pm 11) \times 10^{-4} & (NA31) \\ (-5 \pm 15) \times 10^{-4} & (E731) \end{cases} \tag{7}$$

are in some disagreement and will not be imposed as restrictions on the mass matrices.

To begin our search, we define the 3-family mass matrices in the weak basis as follows:

$$\mathbf{M}^U = \begin{pmatrix} E_1 & A & D \\ A & E_2 & B \\ D^* & B & C \end{pmatrix}, \qquad \mathbf{M}^D = \begin{pmatrix} E_1' & A' & D' \\ A'^* & E_2' & B' \\ D'^* & B'^* & C' \end{pmatrix} \tag{8}$$

Diagonalization of these matrices according to the following unitary transformations

$$\begin{aligned} U\mathbf{M}^U U^\dagger &= \mathbf{D}^U = diag(m_u, -m_c, m_t) \\ U'\mathbf{M}^D U'^\dagger &= \mathbf{D}^D = diag(m_d, -m_s, m_b) \end{aligned} \tag{9a}$$

then identifies the KM mixing matrix to be

$$V_{KM} = UU'^\dagger \tag{9b}$$

which can be expressed in terms of 3 angles and 1 phase for three families. A common phase rotation has been applied to the up and down matrices in (8) to eliminate two unphysical phases. We are then left with 16 parameters (12 magnitudes and 4 phases) to explain the 10 independent physical quantities (6 quark masses and 3 KM angles and 1 phase). Obviously for pure predictive power, one would like to reduce the 16 parameters to less than 10. The original Fritzsch ansatz[3] has just 8 free parameters, as E_1, E_2, D, E_1', E_2' and D' are set equal to zero on the assumption of nearest-neighbor radiative corrections. But in our search for an upper bound on the top quark mass, we wish to scan the whole 16 parameter space.

In carrying out our search, we shall follow the lead of Fritzsch and work in the weak basis with the hierarchical chiral symmetry-breaking assumption taken into account.

This implies that the $\mathbf{M}^U$ matrix elements are ordered according to

$$0 \lesssim |E_1| \ll |A|, |D|, |E_2| \ll |B| \ll C \tag{10}$$

and similarly for $\mathbf{M}^D$. Whereas Fritzsch also assumed nearest-neighbor interactions, we shall relax that assumption. After finding satisfactory mass matrices, one can then rotate both $\mathbf{M}^U$ and $\mathbf{M}^D$ by the same unitary transformation to bring them into somewhat different forms, as suggested in the superstring framework, for example.

In our previous analysis[6] with Lindner of the hierarchical chiral symmetry-breaking pattern, we noted that at the rank 2 level E_2 should be small relative to B and C and set it equal to zero, as suggested by a seesaw mass-generating mechanism. It follows mainly from this assignment, and in particular for E_2', that the top mass is bounded to be less than about 100 GeV. More generally, we now observe that in the rank 2 case with only C, B and E_2 different from zero, the two invariant trace conditions

$$Tr\ \mathbf{M}^U = C + E_2 = m_t - m_c \tag{11a}$$

$$Tr\ (\mathbf{M}^U)^2 - (Tr\ \mathbf{M}^U)^2 = 2(B^2 - CE_2) = 2m_t m_c \tag{11b}$$

lead to

$$-m_c \leq E_2 \leq m_t \tag{12a}$$

and similarly for $\mathbf{M}^D$,

$$-m_s \leq E_2' \leq m_b \tag{12b}$$

For the ranges

$$0 \leq E_2 \lesssim 3m_c, \qquad -m_s \leq E_2' < 0 \tag{13}$$

we find that top quark masses greater than 100 GeV are possible.

We now return to the full rank 3 matrices and make the following observations that follow from attempting to fit the KM matrix and all the mixing parameters.

The Cabibbo angle generally fixes the phase of A' to lie in the range of $80^o - 90^o$. The phase of B' lies between $\pm 20^o$, and the KM-allowed physical region defines an annular ring in the $\phi_{B'}$ *vs.* m_t plane as in the Fritzsch case.[4] The diagonal elements E_2 and E_2' must lie in the ranges quoted in (13) to give top masses greater than 100 GeV. The magnitude of D' largely controls the ratio $|V_{ub}/V_{cb}|$. If D' is set equal to zero as in the Fritzsch case, we generally find $|V_{ub}/V_{cb}| \lesssim 0.06$. A non-zero value of D' leads to larger values for $|V_{ub}/V_{cb}|$ in somewhat better agreement with (6). The parameters E_1 and E_1' are constrained to be extremely small and have little effect in determining any of the physical variables, aside from the u and d quark masses.

In carrying out a general search for high top mass solutions, we have found the computer time considerably reduced if we consider just the rank-2 case for $\mathbf{M}^U$ with only D, B and C different from zero, which of course implies that $m_u = 0$. This follows the recent interesting suggestion by Nilles, Olechowski and Pokorski[17] that the up quark mass is only generated at the weak scale by instanton effects. Extension to the full rank-3 case can then be treated as a perturbation of the above, and we find only small changes in the results.

With the above procedure, we have allowed $E_2', |D'|, D/B$ and the phases $\phi_{A'}, \phi_{B'}$ and $\phi_{D'}$ to vary over suitable ranges and then calculate the magnitudes $C, B, C', |B'|$ and $|A'|$ by the invariant trace and determinant conditions. Scatter plots of the J-value; $B_d^o - \bar{B}_d^o$ mixing combination $m_t^2|V_{td}^* V_{tb}|^2 R$; $|V_{ub}/V_{cb}|$; and the K meson bag parameter, B_K, *vs.* m_t are then presented in Fig. 1 for solutions which satisfy the KM matrix (2) to within one standard deviation accuracy and simultaneously fall within the ranges quoted in (3) - (6). We see that the largest top mass allowed with one Higgs doublet is found to be about 200 GeV. With equal probability assigned to each point, the most probable top mass appears to lie in the region of 130 GeV, as deduced by Ellis and Fogli[18] on the basis of the Z boson mass, M_W/M_Z ratio and radiative corrections to deep inelastic neutrino scattering. It is especially clear from Fig. 1(c) and 1(d) that the maximum top mass decreases as the lower limits on $|V_{ub}/V_{cb}|$ and B_K are raised.

With two Higgs doublets, one giving mass to the up quarks and the other to the down quarks, equal vacuum expectation values and a charged Higgs mass of 50 (90) GeV, the maximum top mass is lowered considerably and restricted to be less than about 125 (135) GeV. The most probable value occurs around 90 (100) GeV for these two cases. This can be understood on the basis that the charged Higgs scalar contributes to the $B_d^o - \bar{B}_d^o$ mixing box diagrams as well as W boson exchange and that contribution is constructive. With the higher-order contribution to R in Eq. (5b) now being greater than unity, the running top mass m_t must be smaller than before with only W exchange contributions.

We have also searched for any special forms of the mass matrices that may be of interest. One such form was found which has the phase structure originally espoused by Shin[10] for the Fritzsch matrices on the basis of maximal CP violation in the weak interaction sector: all elements are real, except two in the down matrix which are pure imaginary. The forms are

$$\mathbf{M}^U = \begin{pmatrix} 0 & A & A \\ A & A & B \\ A & B & C \end{pmatrix}, \qquad \mathbf{M}^D = \begin{pmatrix} 0 & iA' & -A' \\ -iA' & -A' & B' \\ -A' & B' & C' \end{pmatrix} \qquad (14a)$$

These matrices apply for only a special set of parameters and correspond to a top mass of 135 GeV in the minimal Higgs model with the respective points indicated by large black dots in Fig. 1, and for a top mass of 120 GeV with two Higgs doublets and a charged Higgs mass of 50 GeV.

Finally we note that Casas and Muñoz[11] have studied a generic ansatz for the quark mass matrices which they claim emerges naturally in superstring orbifold compactifications. The conditions on the mass matrices in terms of our notation in (8) are

$$|E_1|,\ |A|,\ |D|,\ |B|\ <<\ |E_2|\ <<\ C, \qquad |E_1'|,\ |A'|,\ |D'|,\ |B'|\ <<\ |E_2'|\ <<\ C' \quad (15)$$

We can attempt to rotate both $\mathbf{M}^U$ and $\mathbf{M}^D$ into this form, while leaving V_{KM} unchanged, by applying a common unitary transformation to both matrices according to

$$\mathbf{M}'^U = U''\mathbf{M}^U U''^\dagger, \qquad \mathbf{M}'^D = U''\mathbf{M}^D U''^\dagger \quad (16)$$

In particular, U'' should serve nearly to diagonalize both $\mathbf{M}^U$ and $\mathbf{M}^D$. For purposes of illustration, let us apply this procedure to the special matrices in (14a). As a typical example we find

$$\mathbf{M}'^U = \begin{pmatrix} 0 & 0.07 & 0.08 \\ 0.07 & -1.36 & -0.11 \\ 0.08 & -0.11 & 220 \end{pmatrix}, \qquad \mathbf{M}'^D = \begin{pmatrix} 0 & 0.027e^{i85^\circ} & 0.027e^{i95^\circ} \\ 0.027e^{-i85^\circ} & -0.10 & -0.25 \\ 0.027e^{-i95^\circ} & -0.25 & 5.29 \end{pmatrix} \quad (14b)$$

i.e., we can not rotate *both* $\mathbf{M}^U$ and $\mathbf{M}^D$ into the nearly diagonal form with the ordering suggested by (15) without changing the KM matrix itself. This appears to be a general feature of the whole class of solutions we have found and suggests that more than one additional stage of gauge-symmetry breaking after compactification may be required.

In summary, we have performed a general search for 3-family mass matrices which fit the KM matrix to within one standard deviation accuracy and satisfy additional constraints on CP violation, $B_d^o - \bar{B}_d^o$ mixing, $|V_{ub}/V_{cb}|$ and the K meson bag parameter. Our main conclusions are that with the standard minimal Higgs structure, solutions exist for top masses in the range $80 \lesssim m_t \lesssim 200$ GeV with the most probable value centered around 130 GeV, in agreement with the radiative correction analysis of Ref. 18. In the presence of two Higgs doublets and a charged Higgs scalar of mass 50 (90) GeV, the upper bound on the top mass is lowered to 125 (135) GeV.

This strongly suggests that if top is found with a mass of 150 GeV or larger, the two Higgs doublet version with equal VEV's can be ruled out, at least in the 3-family framework, unless the charged Higgs scalar is much larger than 100 GeV. At this point there is no necessity to invoke a fourth family of massive quarks to explain all the mixing data.

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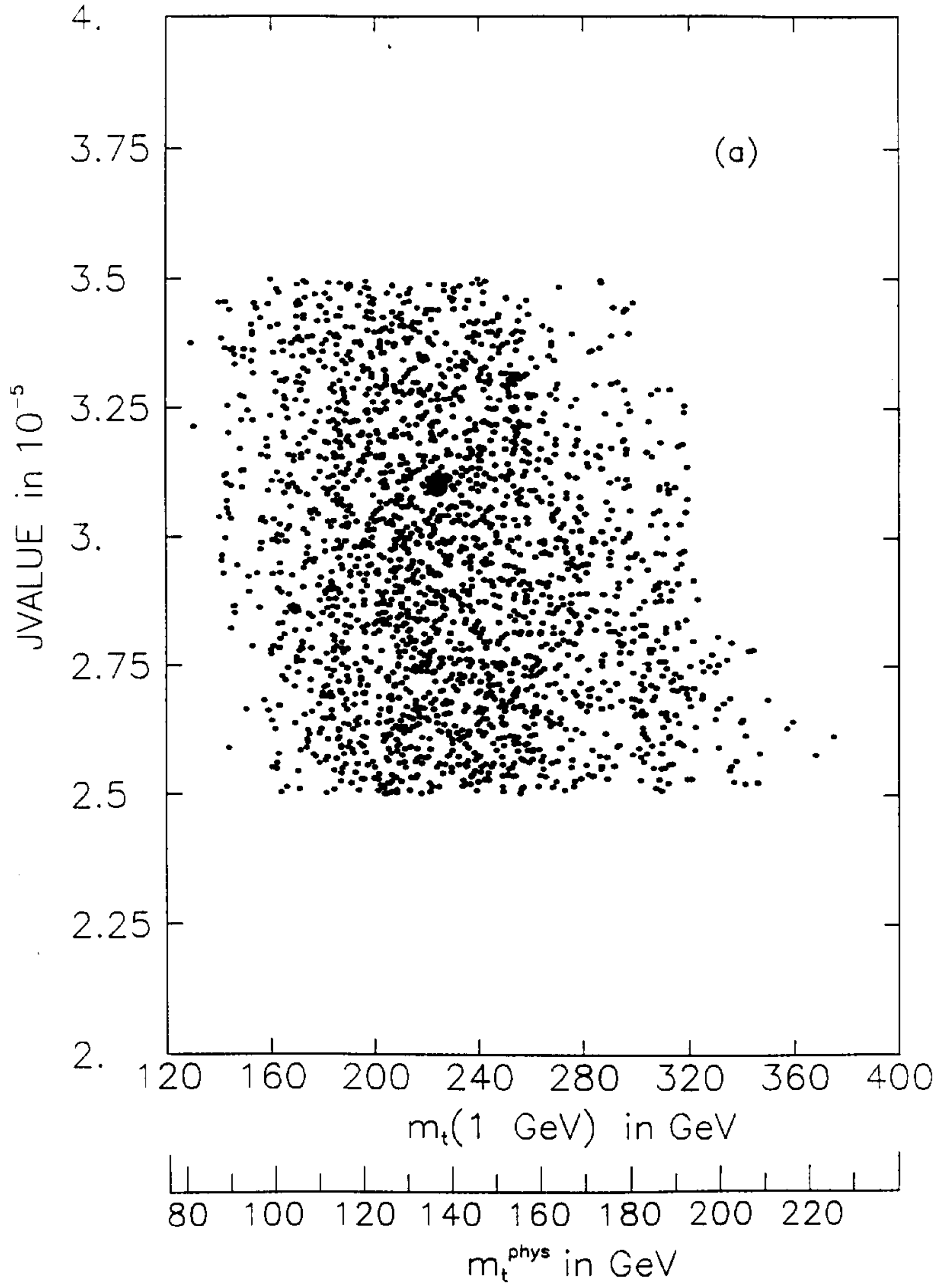
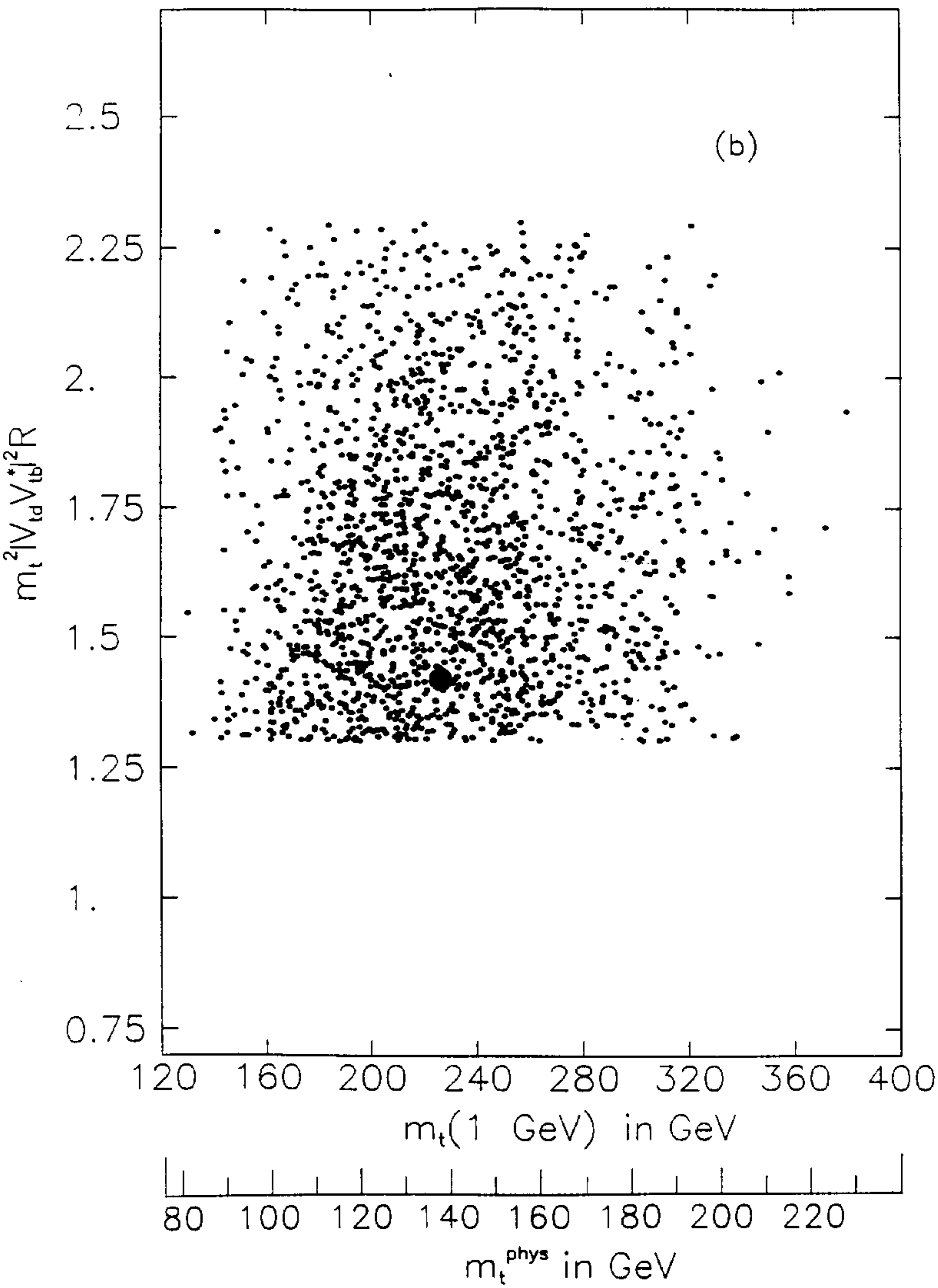
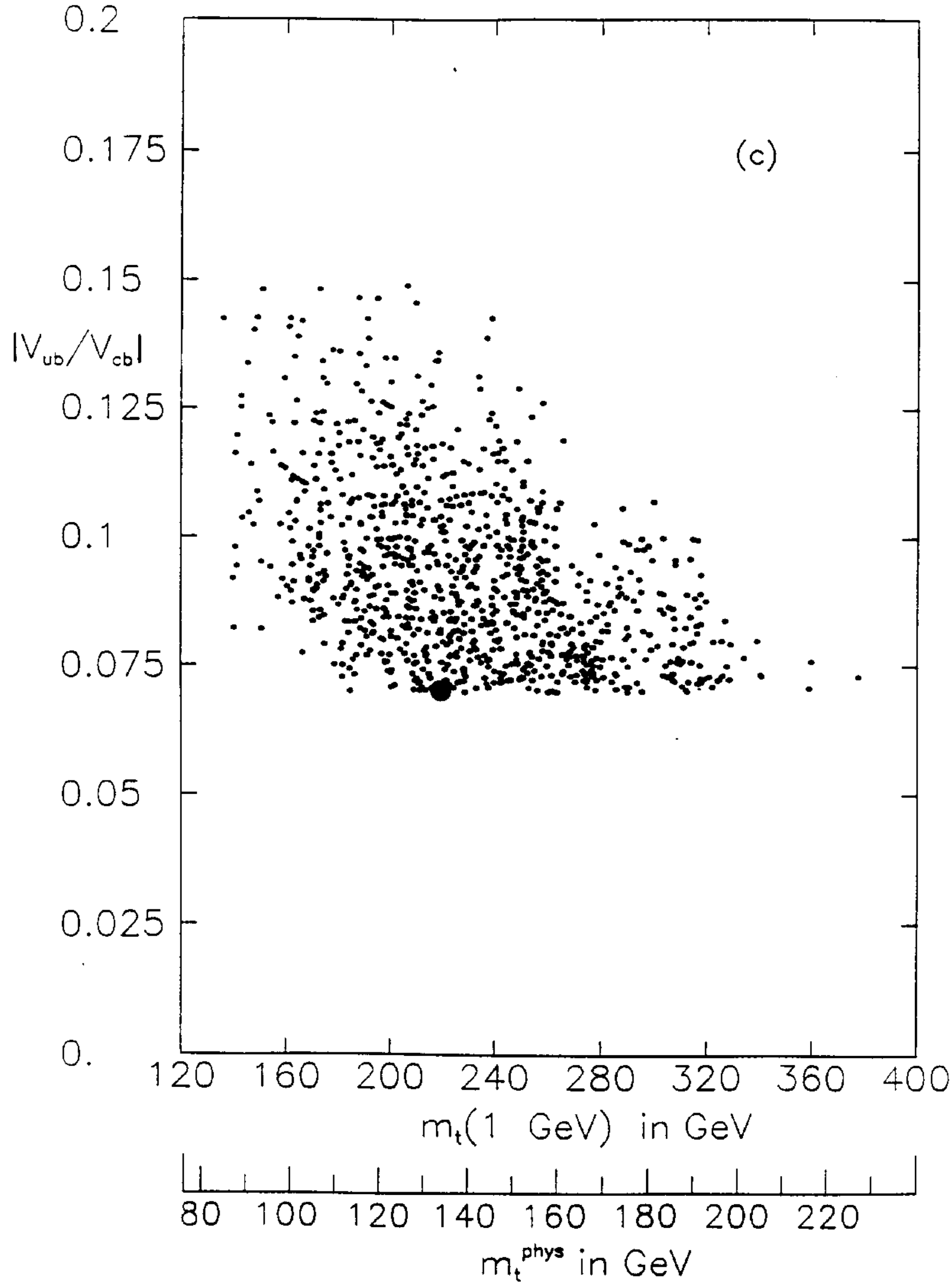
Figure Caption

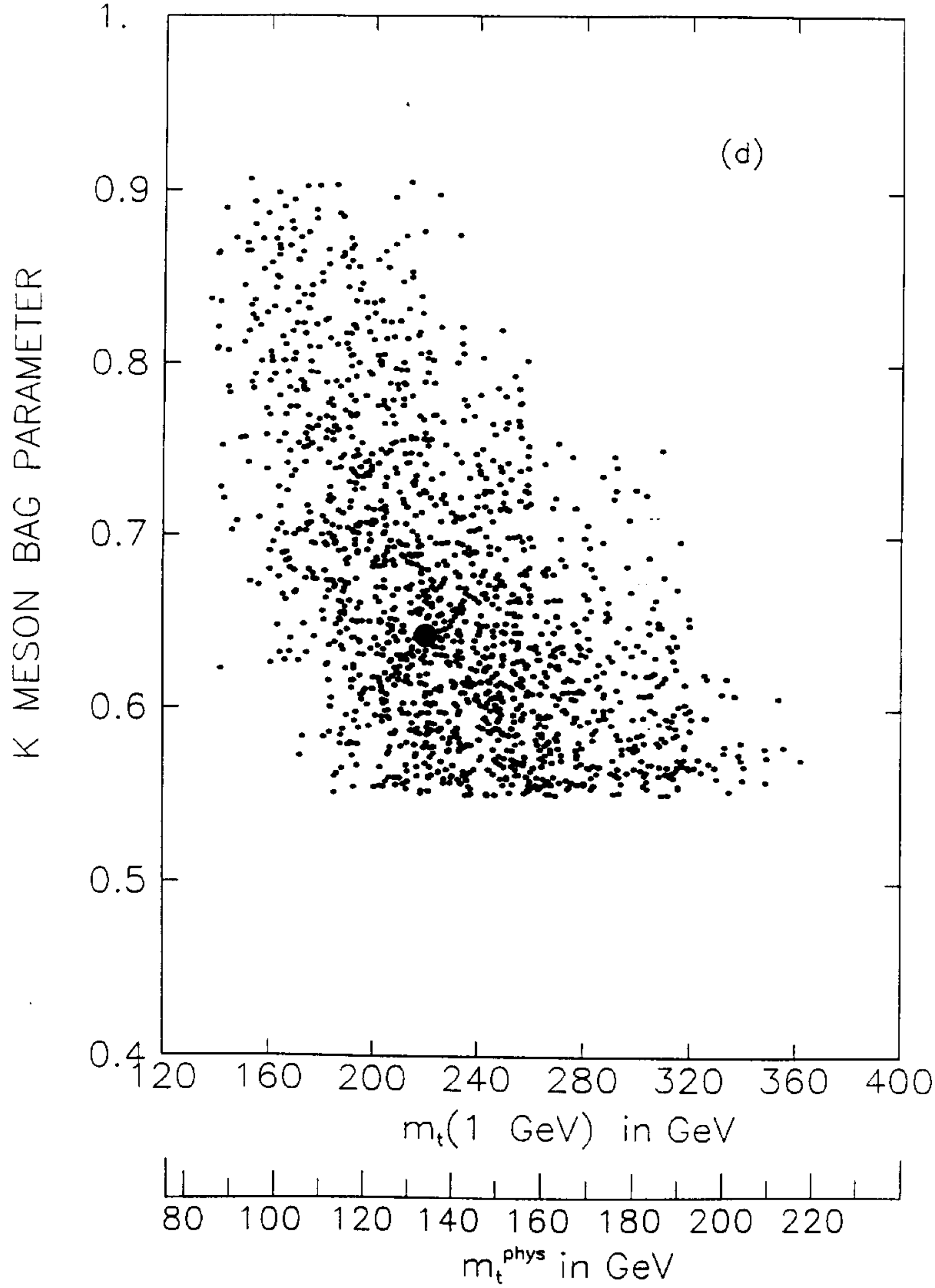
Figure 1: Scatter plots of (a) the J-value for CP violation, (b) the parameter combination appearing in $B_d^o - \bar{B}_d^o$ mixing, (c) $|V_{ub}/V_{cb}|$ and (d) the K meson bag parameter *vs.* the top quark mass for mass matrix solutions which satisfy the criteria stated in the text. The large black dots refer to the special case of Eq. (14).



(a)
JVALUE in 10^{-5}
4.
3.75
3.5
3.25
3.
2.75
2.5
2.25
2.
120 160 200 240 280 320 360 400
m_t(1 GeV) in GeV
80 100 120 140 160 180 200 220
m_t^{phys} in GeV


(b)
$m_t^2|V_{td}V_{tb}^*|^2R$
2.5
2.25
2.
1.75
1.5
1.25
1.
0.75
120 160 200 240 280 320 360 400
$m_t(1\ \text{GeV})$ in GeV
80 100 120 140 160 180 200 220
m_t^{phys} in GeV


(c)
$|V_{ub}/V_{cb}|$
0.2
0.175
0.15
0.125
0.1
0.075
0.05
0.025
0.
120 160 200 240 280 320 360 400
$m_t(1\ \mathrm{GeV})$ in GeV
80 100 120 140 160 180 200 220
m_t^{phys} in GeV



(d)
K MESON BAG PARAMETER
1.
0.9
0.8
0.7
0.6
0.5
0.4
120 160 200 240 280 320 360 400
$m_t(1\ \text{GeV})$ in GeV
80 100 120 140 160 180 200 220
m_t^{phys} in GeV